


# SOALAN RAMALAN

## *MATEMATIK TAMBAHAN*

## *KERTAS 1*

**SUKATAN MEMBULAT**
*CIRCULAR MEASURE*

**SIR VEN**
*(GURU ADIWIRA KEBANGSAAN 2019)*

**SPM 2023**

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Diagram 7 shows two sectors $AOD$ and $BOC$ of two concentric circles with centre $O$.
*Rajah 7 menunjukkan dua buah sektor AOD dan BOC bagi dua bulatan dengan pusat sepunya O.*

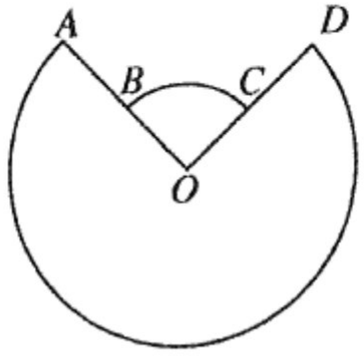



Diagram 7
*Rajah 7*

The angle subtended at the centre $O$ by the major arc $AD$ is $7\alpha$ radians and the perimeter of the whole diagram is 50 cm.
Given $OB = r$ cm, $OA = 2OB$ and $\angle BOC = 2\alpha$, express $r$ in terms of $\alpha$.
*Sudut yang dicangkum pada pusat O oleh lengkok major AD ialah $7\alpha$ radian dan perimeter seluruh rajah ialah* 50 cm.
*Diberi $OB = r$ cm, $OA = 2OB$ dan $\angle BOC = 2\alpha$, ungkapkan r dalam sebutan $\alpha$.*

[3 *marks*]
[3 *markah*]

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Diagram *5 shows a circle with centre O.*
*Rajah 5 menunjukkan sebuah bulatan dengan pusat O.*

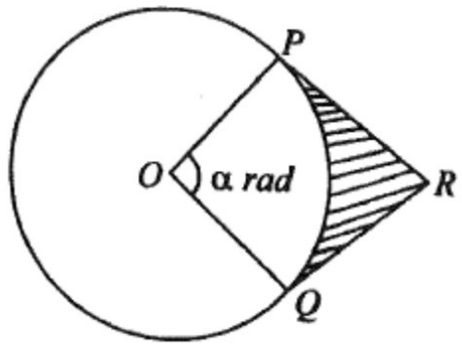



Diagram 5
*Rajah 5*

*PR* and *QR* are tangents to the circle at points *P* and *Q* respectively. It is given that the length of minor arc *PQ* is 4 cm and $OR = \frac{5}{\alpha}$ cm.
*PR dan QR masing-masing adalah tangen kepada bulatan itu pada titik P dan titik Q.*
*Diberi bahawa panjang lengkok minor PQ ialah* 4 cm *dan* $OR = \frac{5}{\alpha}$ cm.

Express in terms of α
*Ungkapkan dalam sebutan* α

(*a*) the radius, *r*, of the circle,
*jejari, r, bulatan itu,*

(*b*) the area, *A*, of the shaded region.
*luas, A, kawasan berlorek.*

Diagram ***4 shows a rhombus*** *OACB* inscribed in sector *AOB* with centre *O* and radius *r* cm.
*Rajah* 4 *menunjukkan sebuah rombus OACB yang terterap dalam sektor AOB dengan pusat O dan jejari r* cm.

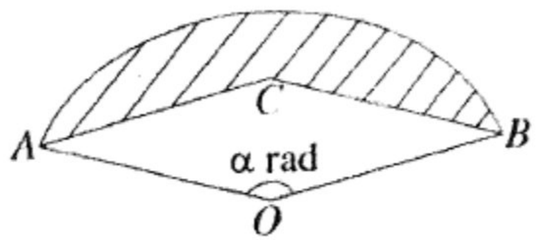



Diagram 4
*Rajah* 4

Given the area of sector *AOB* is 18 $cm^2$, express
*Diberi luas sektor AOB ialah* 18 $cm^2$, *ungkapkan*

(*a*) $\alpha$ in terms of *r*,
$\alpha$ *dalam sebutan r,*

(*b*) the perimeter, in cm, of the shaded region in terms of *r*.
*Perimeter, dalam* cm, *bagi kawasan berlorek dalam sebutan r.*

[3 *marks*]

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Mathematics Society of SMK Muhibah organised a competition to design a logo for the society.
*Persatuan Matematik SMK Muhibah menganjurkan satu pertandingan mencipta logo untuk persatuan itu.*


Diagram 3
*Rajah* 3

Diagram 3 shows the circular logo designed by Amar. The three blue coloured regions are congruent. It is given that the perimeter of the blue colour region is $20\pi$ cm.
*Rajah 3 menunjukkan logo berbentuk bulatan yang dicipta oleh Amar. Ketiga-tiga kawasan berwarna biru adalah kongruen. Diberi bahawa perimeter bagi kawasan berwarna biru ialah* $20\pi$ cm.
[Use / *Guna* $\pi = 3.142$]
Find
*Cari*

(*a*) the radius, in cm, of the logo to the nearest integer, [3 *marks*]
8.2 *jejari, dalam* cm, *bagi logo itu kepada integer terhampir,* [3 *markah*]

(*b*) the area, in $cm^2$, of the yellow coloured region. [4 *marks*]
8.3 *luas, dalam* $cm^2$, *bagi kawasan yang berwarna kuning.* [4 *markah*]

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Susan wants to make a cap in the shape of a cone as shown in Diagram 6.1. The height of the cone is 24 cm.
*Susan ingin menghasilkan topi berbentuk kon seperti yang ditunjukkan dalam Rajah* 6.1. *Tinggi kon itu ialah* 24 cm.

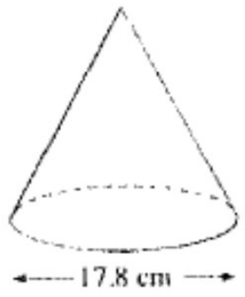



Diagram 6.1
*Rajah* 6.1

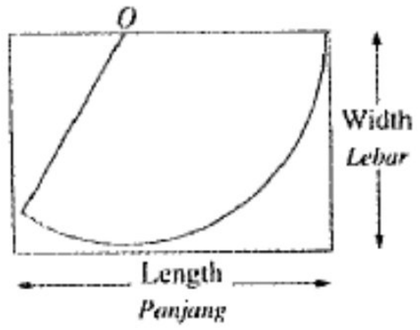



Diagram 6.2
*Rajah* 6.2

Diagram 6.2 shows the net of the cone in the form of sector of a circle with centre $O$ drawn on a rectangular card.
*Rajah* 6.2 *menunjukkan bentangan kon yang berbentuk sektor bagi sebuah bulatan dengan pusat* $O$ *yang dilukis di atas sekeping kad berbentuk segi empat tepat.*

(*a*) Calculate the minimum value, in cm, of length and of width of the card to the nearest integer.
*Hitung nilai minimum, dalam* cm, *bagi panjang dan lebar kad itu kepada integer terdekat.*

[5 *marks*]
[5 *markah*]

(*b*) Hence, find the area, in $cm^2$, of the card unused.
*Seterusnya, cari luas, dalam* $cm^2$, *kad yang tidak digunakan.*

[3 *marks*]
[3 *markah*]

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Diagram 24 shows the shot target board in square shape with sides of 15 cm in the shooting range. There is a black circle with a diameter of 5 cm in the middle of the target board..

*Rajah 24 menunjukkan papan sasaran tembakan berbentuk segi empat sama bersisi 15 cm yang terdapat di lapang sasar. Terdapat sebuah bulatan hitam berdiameter 5 cm di tengahnya.*

PAHANG - 2018

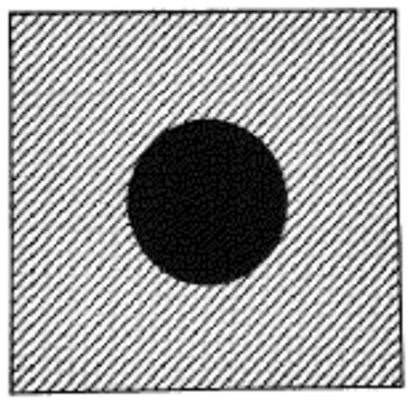


Diagram 24 / *Rajah* 24

Chong is a sharpshooter and his shots are always on target board. Find the probability, Chong shot on the shaded area.

*Chong merupakan seorang penembak tepat yang tembakannya sentiasa mengenai papan sasaran tembakan. Cari kebarangkalian, tembakan Chong mengenai kawasan berlorek.* [Use / *Gunakan* $\pi = 3.142$]

[3 marks]

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Diagram 8 shows six concentric circles. The largest circle has a radius of $r$ cm. The radius of each subsequent circle is five over six of the radius of its previous one. The areas of the circles form a geometric progression in descending order.

*Rajah 8 menunjukkan enam bulatan konsentrik. Bulatan terbesar mempunyai jejari r cm. Panjang jejari setiap bulatan yang berikut adalah lima perenam daripada panjang jejari bulatan sebelumnya. Luas bulatan-bulatan itu membentuk suatu janjang geometri dalam susunan menurun.*

Diagram 8 / *Rajah 8*

Find / *Cari*

(a) the common ratio,
*beza sepunya,*

(b) the area of the first circle given the sum of the total areas of the first three circles is 17.41 cm².
*luas bulatan pertama jika diberi jumlah luas tiga bulatan yang pertama ialah* 17.41 cm².

[4 *marks / markah*]

Diagram 17 shows two sectors, $MON$ and $POQ$, of two concentric circles with centre $O$.

*Rajah 17 menunjukkan dua sektor, MON dan POQ, bagi dua bulatan dengan pusat sepunya O.*

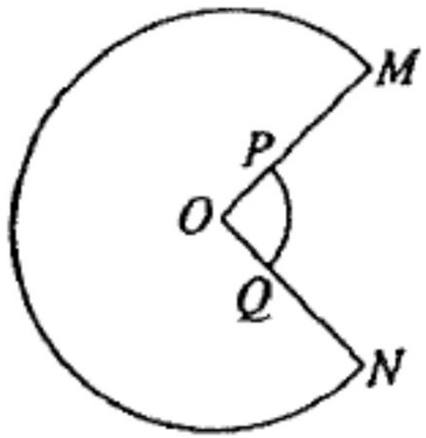



Diagram 17/*Rajah 17*

The angle subtended at the centre $O$ by the major arc $MN$ is $9x$ radians and the perimeter of the whole diagram is 38 cm. Given that $OP = r$, $OM = 3OP$ and $\angle POQ = 3x$ radians, express $r$ in terms of $x$.

*Sudut yang dicangkum pada pusat O oleh lengkok major MN ialah* $9x$ *radian dan perimeter seluruh rajah itu ialah* 38 cm. *Diberi* $OP = r$, $OM = 3OP$ *dan* $\angle POQ = 3x$ *radian, ungkapkan r dalam sebutan x.*

[3 *marks*/3 *markah*]

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Diagram 16 shows two sectors $POQ$ and $ROS$ of two concentric circles with centre $O$.
*Rajah 16 menunjukkan dua buah sektor POQ dan ROS bagi dua bulatan dengan pusat sepunya O.*

The angle subtended at the centre $O$ by the major arc $PQ$ is $9\beta$ radians and the perimeter of unshaded area is 65 cm. Given $OR = r$ cm, $OP = 2OR$ and $\angle ROS = 3\beta$ radians, express $r$ in terms of $\beta$ and $\pi$.
*Sudut yang dicangkum pada pusat O oleh lengkok major PQ ialah $9\beta$ radian dan perimeter bahagian yang tidak berlorek ialah 65 cm. Diberi $OR = r$ cm, $OP = 2OR$ dan $\angle ROS = 3\beta$ radian ungkapkan r dalam sebutan $\beta$ dan $\pi$.*

[3 *marks / markah*]

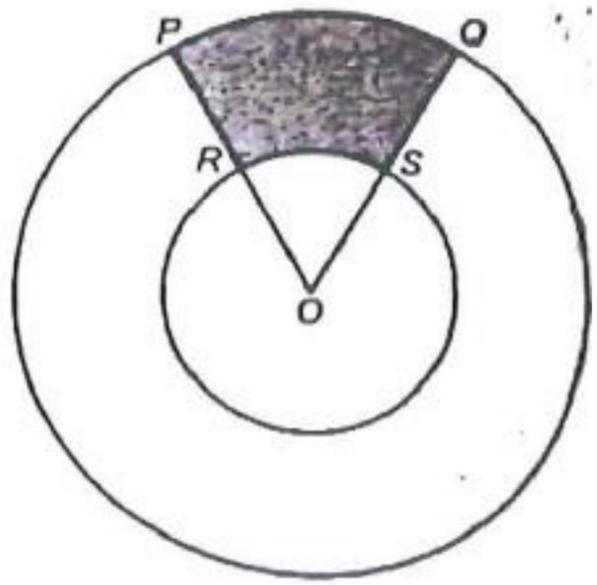


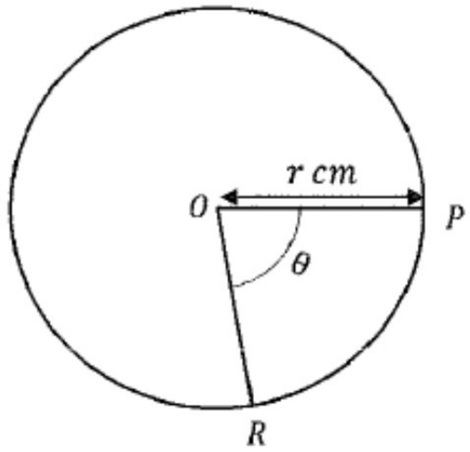

Diagram 16 / *Rajah* 16

Answer / *Jawapan* :

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Rajah 4 menunjukkan sebuah bulatan berpusat O dan berjejari $r$ cm. Diberi sudut $POR=\theta$ radian.

*Diagram 4 shows a circle, with centre O and radius r* cm. *Given the angle of* $POR=\theta$ *radians.*




Rajah 4 / *Diagram* 4

(a) Diberi luas sektor major POR ialah 5 kali luas sektor minor POR, cari nilai $\theta$.
*Given the area of the major sector POR is 5 times the area of the minor sector POR, find the value of* $\theta$.

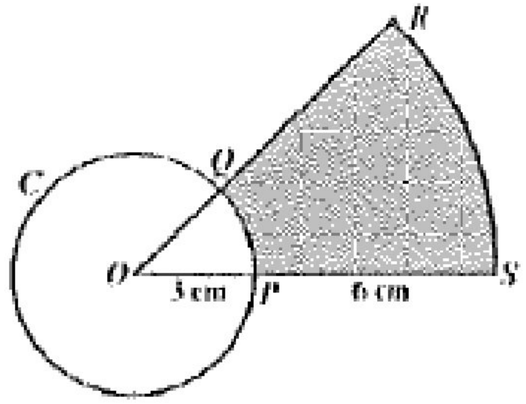
[2 *markah*]
[2 *marks*]

(b) Diberi perimeter sektor minor POR ialah 15 cm. Cari nilai $r$, betul kepada 3 tempat perpuluhan. [Gunakan $\pi = 3.142$]
*Given the perimeter of the minor sector POR is* 15 cm. *Find the value of r, correct to* 3 *decimal places.* [*Use* $\pi = 3.142$]

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Rajah 9 menunjukkan sebuah bulatan $CPQ$ berpusat pada $O$ dengan jejari 3 cm. Garis $OP$ dan garis $OQ$ dipanjangkan ke $S$ dan $R$ masing-masing. $ORS$ adalah sektor bulatan berpusat pada $O$. Diberi bahawa $PS = 6$ cm dan luas kawasan berlorek adalah sama dengan luas bulatan $CPQ$.

*Diagram 9 shows a CPQ circle centered at O with a radius of 3 cm. The line OP and the line OQ are extended to S and R respectively. ORS is a sector of a circle centered at O. Given that PS = 6 cm and the area of the shaded region is equal to the area of the circle CPQ.*




Rajah 9 / *Diagram 9*

(a) Tunjukkan $\angle POQ = \frac{1}{4}\pi$ rad.

*Show that* $\angle POQ = \frac{1}{4}\pi$ rad.

(b) Cari perimeter kawasan berlorek. [Gunakan $\pi = 3.142$ rad]
*Find the perimeter of the shaded region.* [Use $\pi = 3.142$ rad]

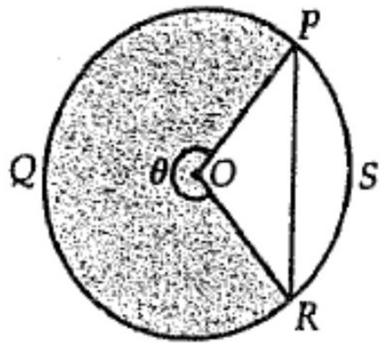
**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Rajah 13 menunjukkan pandangan atas sebuah kolam berbentuk bulatan *PQRS*. Ia mempunyai tangga berbentuk segitiga yang turun ke dasar kolam dan membentuk sektor bulatan *OPSR*. Diberi panjang lengkok *PQR* ialah 31.5 cm dan luas kawasan berlorek ialah $118\frac{1}{8}$ cm².

[Guna π = 3.142 rad]

*Diagram* 13 *shows a top view of a circular pond PQRS. It has a triangular shaped staircase that descends to the bottom of the pool and forms a circular sector of the OPSR. Given that the length of the arc PQR is* 31.5 cm *and the area of the shaded region is* $118\frac{1}{8}$ cm².

[*Use* π = 3.142 rad]




Rajah 13/ *Diagram* 13

Hitung
*Calculate*

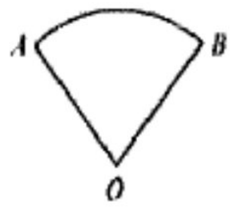
(a) jejari, dalam cm, bagi bulatan PQRS,
*radius, in* cm, *of a circle PQRS,*
[2 *markah*]
[2 *marks*]

b) $\theta$, dalam radian,
$\theta$, *in radians*,
[2 *markah*]
[2 *marks*]

c) luas, dalam cm², tembereng *PRS*.
*area, in* cm², *the segment of PRS*.
[4 *markah*]
[4 *marks*]

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

(a) Rajah 4(*a*) menunjukkan panjang lengkok AB ialah 1.5 kali panjang jejarinya, *OA*.
*Diagram* 4(*a*) *shows the length of arc AB is* 1.5 *times its radius, OA.*

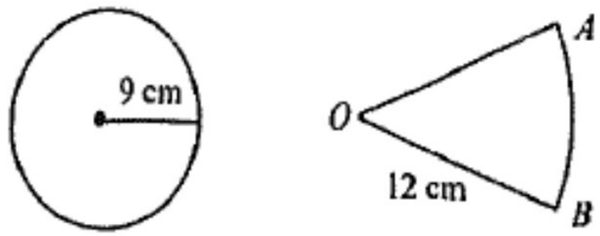



Rajah 4(*a*) / *Diagram* 4(*a*)

Cari sudut *AOB* dalam radian.
*Find the angle of AOB in radian.*

[2 marks]
[2 mar

(b) Rajah 4(*b*) menunjukkan seutas dawai yang berbentuk bulatan dengan jejari 9 c
dibengkokkan untuk membentuk sebuah sektor *AOB* dengan pusat *O* dan berjejari 12 cm.
*Diagram* 4(*b*) *shows a wire in the form of a circle with radius 9 cm is bent to form a seq*
*AOB with centre O and radius* 12 cm.



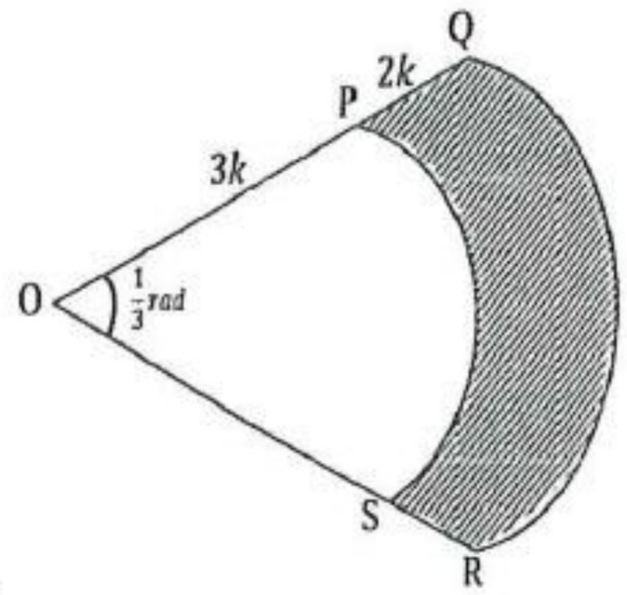

Rajah 4(*b*) / *Diagram* 4(*b*)

Cari sudut bagi sektor *AOB* dalam radian. [ guna $\pi = 3.142$ ]
*Find the angle of sector AOB in radian.* [ *use* $\pi = 3.142$ ]

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Rajah 10 menunjukkan dua sektor *OPS* dan *OQR* bersudut $\frac{1}{3}$ rad, berpusatkan di *O*.

*Diagram* 10 *shows two sectors OPS and OQR have angle* $\frac{1}{3}$ rad, *with centre O.*




Rajah 10 / *Diagram* 10

(a) Cari nilai *k*, jika luas kawasan berlorek PQRS ialah 24 cm$^2$.
*Find the value of k if the area of the shaded region PQRS is* 24 cm$^2$.

(b) Berdasarkan nilai *k* di (*a*), hitung perimeter kawasan berlorek.
*Based on the value ok k in* (*a*). *calculate the perimeter of the shaded region.*

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Length of major arc $AOD$
$= 2r \times 7\alpha$
$= 14r\alpha$
Length of minor arc $BOC$
$= r \times 2\alpha$
$= 2r\alpha$
Perimeter of whole diagram = 50

$$14r\alpha + 2r\alpha + r + r = 50$$
$$16r\alpha + 2r = 50$$
$$8r\alpha + r = 25$$
$$r(8\alpha + 1) = 25$$
$$r = \frac{25}{8\alpha + 1}$$

(*a*) $s_{PQ} = 4$
$r\alpha = 4$
$r = \frac{4}{\alpha}$ cm

(*b*) $PR = \sqrt{\left(\frac{5}{\alpha}\right)^2 - \left(\frac{4}{\alpha}\right)^2}$

$PR = \sqrt{\frac{9}{\alpha^2}} = \frac{3}{\alpha}$
$A$ = Area of shaded region
= Area of quadrilateral $OPRQ$ – Area of sector $OPQ$
$= 2(\text{Area of } \Delta OPR) - \frac{1}{2}r^2\theta$
$= 2\left[\frac{1}{2} \times \frac{3}{\alpha} \times \frac{4}{\alpha}\right] - \left[\frac{1}{2} \times \left(\frac{4}{\alpha}\right)^2 \times \alpha\right]$
$= \frac{12}{\alpha^2} - \frac{8}{\alpha}$
$= \frac{12 - 8\alpha}{\alpha^2}$ cm$^2$

(*a*) Area of sector $AOB$ = 18 cm$^2$

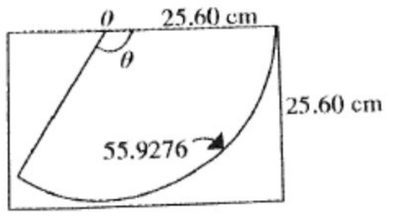
$$\frac{1}{2}r^2\alpha = 18$$
$$\alpha = \frac{36}{r^2}$$

(*b*) Length of arc $AB = r\alpha$

$$= r\left(\frac{36}{r^2}\right)$$
$$= \frac{36}{r}$$

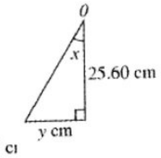
Perimeter of shaded region $= \frac{36}{r} + AC + CB$
$= \frac{36}{r} + r + r$
$= \frac{36}{r} + 2r$

(*a*) $x^2 = 24^2 + 8.9^2$
$x = \sqrt{24^2 + 8.9^2} = 25.60$ cm
Minimum width of the card is 26 cm.




Circumference of the base of the cone = $2\pi r$
$= 2 \times 3.142 \times 8.9$ cm
= 55.9276 cm
$s = r\theta$
$55.9276 = r\theta$
$\theta = \frac{55.9276}{25.6} = 2.185$ radian

$x + \frac{\pi}{2} = \theta$
$x = 2.185 - \frac{\pi}{2}$
= 0.614 rad




$\frac{y}{25.60} = \tan x$
$y = 18.045$ cm
Minimum length
= 25.60 cm + 18.045 cm
= 40.340 cm ≈ 41 cm

(*b*) Area of card = 26 cm × 41 cm
= 1 066 cm$^2$
Area of sector $= \frac{1}{2} \times 25.60^2 \times 2.185$
= 715.98 cm$^2$
Area of unused card = 1 066 cm$^2$ – 715.98 cm$^2$
= 350.02 cm$^2$

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

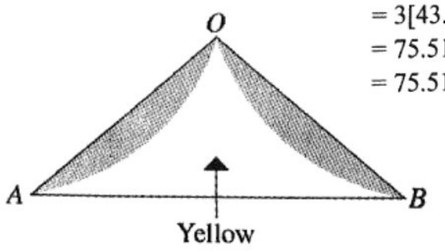
(a) $6 \text{ arcs} = 20\pi$

$6r\theta = 20?$

$6r\left[60° \times \frac{\pi}{180°}\right] = 20\pi$

$r = 10 \text{ cm}$

(b)




Area of yellow coloured region

= 3[area of triangle $OAB$] – 6[area of segment]

$= 3\left[\frac{1}{2}(10)^2 \sin 120°\right] - 6\left[\frac{1}{2}(10)^2\left(60° \times \frac{3.142}{180°}\right) \frac{1}{2}(10)^2 \sin 60°\right]$

$= 3[43.3013] - 6[52.3667 - 43.3013]$

$= 75.5115$

$= 75.51 \text{ cm}^2$

Luas = $6.283 \text{ cm}^2$ or $2\pi \text{ cm}^2$

**B2:** $\frac{1}{2}(7)^2(0.5236^*) - \frac{1}{2}(5)^2(0.5236^*)$ or $\frac{1}{2}(7)^2(\frac{\pi}{6}^*) - \frac{1}{2}(5)^2(\frac{\pi}{6}^*)$

**B1:** $\frac{1}{2}(7)^2(0.5236^*)$ *or* $\frac{1}{2}(5)^2(0.5236^*)$ or $\frac{1}{2}(7)^2(\frac{\pi}{6}^*)$ *or* $\frac{1}{2}(5)^2(\frac{\pi}{6}^*)$

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$r, \frac{5}{6}r, \frac{25}{36}r, \ldots$

(a) Common ratio $= \frac{\frac{5}{6}r}{r}$

$= \frac{5}{6}$

(b) Areas of circles

$\pi r^2, \pi\left(\frac{5}{6}r\right)^2, \pi\left(\frac{25}{36}r\right)^2, \ldots$

$S_3 = \frac{a\left[1 - \left(\frac{5}{6}\right)^3\right]}{1 - \frac{5}{6}} = 17.41$

$\frac{91}{36}a = 17.41$

$a = 6.89$

$OM = 3r$

$3r(9x) + r(3x) + 2r + 2r = 38$

$30rx + 4r = 38$

$15rx + 2r = 19$

$r(15x + 2) = 19$

$r = \frac{19}{15x + 2}$

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

Perimeter of unshaded region = 65

$PQ + QS +$ Major Arc $PQ +$ Arc $RS = 65$

$$2r + (2\pi - 9\beta)2r + r(3\beta) = 65$$

$$r(2 + 4\pi - 15\beta) = 65$$

$$r = \frac{65}{2 + 4\pi - 15\beta}$$

$$\frac{1}{2}r^2(2\pi - \theta) = 5(\frac{1}{2}r^2\theta)$$

$$\theta = \frac{\pi}{3}$$

$$r + r + r(\frac{\pi}{3}) = 15$$

$$\theta = 4.922$$

$$\pi(3)^2 = \tfrac{1}{2}(9)^2\theta - \tfrac{1}{2}(3)^2\theta$$

$$\pi(9) = \tfrac{1}{2}(81)\theta - \tfrac{1}{2}(9)\theta$$

$$9\pi = 36\theta$$

$$\theta = \tfrac{1}{4}\pi \text{ rad.}$$

Terbukti.

Perimeter kawasan berlorek:

$$\frac{\pi}{4}(3) + 9 + 9 + \frac{\pi}{4}(9)$$

$$= 3\pi + 18$$

$$= 27.42\ cm$$

$$\frac{1}{2}j^2(\frac{31.5}{j}) = 118.125$$

$$j = 7.5 \text{ cm}$$

$$7.5\theta = 31.5$$

$$\theta = 4.2 \text{ rad}$$

$\theta_2 = 2.084$ rad atau $119.39^o$

$\pi(7.5)^2 - 118.125$

$58.6125 - \frac{1}{2}(7.5)^2 \sin 119.39^o$

$36.028 \text{ cm}^2$

**PENYEMPURNAAN KUASA DUA**
*COMPLETING THE SQUARE*

$s = j\theta$

$1.5x = x\theta$

$\theta = 1.5 \text{ rad}$

$s = j\theta$

$= 9(2\pi)$

$= 18\pi$

$AB = 18\pi - 12 - 12 = 32.556$

$32.556 = 12\theta$

$\theta = 2.713$

$24 = \frac{1}{2}(25k^2)(\frac{1}{3}) - \frac{1}{2}(9k^2)(\frac{1}{3})$

$144 = 16k^2$

$k = 3$

$9(\frac{1}{3}) + 15(\frac{1}{3}) + 12$

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